Personal Thoughts on Mature Teaching

David M. Bressoud

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This book is directed toward new teachers, but much of the advice can be summarized in two fundamental attitudes that are essential for all teachers: enthusiasm for the subject and concern for the students. These attitudes got me through all manner of pedagogical mistakes in my early years as a teacher. As I have matured, I have found that my early enthusiasm and concern are still important, but they are not enough. These are my thoughts about what comes next for those who have been teaching for several years, who know the basics of what to do and what not to do in the classroom, who already are good teachers.

I have always known that enthusiasm is critical. This intuition was confirmed in "The Quest for Excellence in University Teaching" [4] which lists enthusiasm as the most frequently cited attribute of excellent teachers, whether one asks students, colleagues, or administrators. This fact echoes what I have experienced in my own classes and those of others. Enthusiasm is infectious. Even when my students do not share it, they respect it.

My first responsibility in preparing for any class is to find something that I consider interesting and exciting, that I want to show and to share. The fact that I find the material extremely elementary is no excuse, nor that I have taught this course twenty times before, nor even that I was not involved in constructing the syllabus which to me seems disjointed and pointless. As a good teacher, I look for new and interesting examples and illustrations, even in the most elementary mathematics. I search for fresh insights and approaches to what I have taught before and never rely on last year's notes. I seek the threads that tie together the pieces of the syllabus.

One danger of enthusiasm is to become overly ambitious for your class. This is an error that I committed frequently in my early years of teaching; many new teachers do. Even after more than twenty years at the head of the classroom, I still make this mistake more than I would like to admit. It is a forgivable error that has the potential for good—I have seen it stretch students far beyond what they thought they were capable of—provided it is accompanied by the second critical attitude, concern for the student.

Steve Krantz has said a lot of good things about the basic components of concern for your students: respect them and their questions; get to know at least some of them as people; honor the commitment to have times when you

are available to them; learn to be sympathetic, receptive, and patient; be aware of your audience; care. It is common for new instructors to have a rough time the first semester they are in charge of their own class. The assignments and exams are often much too hard. They do not yet have an adequate repertoire of examples and alternate explanations to fall back on when difficulties are encountered. New instructors can be oblivious to some of the pitfalls that students will encounter. But concern for the students will overcome these. I have been privileged to see the student evaluations of several teachers who before that semester had never been responsible for their own class but who had both enthusiasm and concern. Many of the evaluations were critical, describing the things that had gone wrong during the semester. But all of the critical evaluations ended with words to the effect "but she is going to be a really good teacher." They were right.

There comes a time when the naive enthusiasm and concern of the new teacher is not enough. For me it arrived after I had developed confidence in my own ability to teach. I knew the difficulties that students would have and I carefully set out my warnings. I had laid up a stock of illuminating examples that I had seen work for students who were confused. I knew the important concepts that must be stressed and how to tie them together effectively. I was writing assignments that stretched my students, but not beyond their abilities. Exam results were coming in that were not quite what I wanted, but that were adequate. And so I dared to probe a little deeper, to give the students an opportunity to show off their understanding. What I discovered horrified me.

I believe that this moment comes to most of us. It might happen when talking with a student in your office. It might be the result of asking something on a test that goes beyond formulaic responses and requires students to draw on their knowledge to synthesize an answer to a question that is not quite like any they have seen before. It might arrive in a written report in which, for the first time, they are required not just to give the answer—which you know they can find—but to explain how they got that answer. Where you had thought there was understanding, you discover confusion. It seems that they have learned nothing and that your only recourse is to go back to the beginning and start all over again.

This is a moment of crisis—in its literal sense of decision—for the good teacher. I have known those who have resolved it by blaming the students, either their unwillingness to work or the poor preparation that they received from others. I have known those who have resolved it by resigning themselves to the belief that what happens in their classrooms for the majority of students will never be more than a superficial and temporary acquisition of what is needed to pass the course. They decide to focus on those few students who are most like themselves.

Either of these responses marks the beginning of a loss of concern. The concerned response is to seize this opportunity to recognize the depths of our

own ignorance about what actually happens in the classroom and how students learn. If we are good teachers, we will force ourselves to begin exploring the murky waters of pedagogical theory and educational psychology.

This is our opportunity to honestly face what does and does not work in our own teaching and to discover what others have done, to experiment with their ideas and techniques. We have the opportunity to build up a body of insights and ways of capitalizing on those insights. It is my belief that this is a dynamic process that never ends. The truths that I now know about what happens in my classroom are subject to articulation, refinement, and elaboration. Priorities will shift. Techniques for achieving those priorities will always be under trial. Being a good teacher has become a highly personal quest.

Ironically, the great truths that I and others discover are never new. Reading through the literature of education, we find that each hard-won truth was known to the previous generation and to generations before that. But that does not eliminate the need to rediscover these truths for ourselves, individually. We are now seekers after wisdom, and there is no quick route to it. We can learn from the wisdom that others have accumulated, but it is not our own wisdom until we have sifted it, found our own ways of applying it to our teaching, watched it succeed and watched it fail, and made its lessons part of our own story.

I would like to offer some of my own truths. They are neither new nor will they solve your problems. These are not answers. These are places to begin asking your own questions.

1 Teach the students you have

The temptation to blame the attitudes of our students or their preparation from previous teachers is very real because it often seems justified. Working to improve teacher training or what happens in our K-12 schools is an important and worthy contribution, but such efforts do not absolve us from the responsibility to face the students before us with all of their imperfections. We must begin by engaging them where they are. This has nothing to do with abandoning standards or watering down the curriculum. On the contrary, nothing will be accomplished unless we are optimistically realistic about our students and their capabilities.

The situation is never as bad as the cynics and pessimists paint it. I start each semester by asking each student about expectations for the class and what they want to get from it. Almost universally, they want to "understand" the mathematics in this course, and they expect to have to work hard. But they are busy people with many demands on their time, and they are uncertain about what it means to understand mathematics or how to go about accomplishing this. I must provide the structure to help them achieve these goals. Also, no matter how well I may think I know the abilities of the students who will be in my class, there is no substitute for diagnostic tests or assignments.

2 Establish and communicate clear goals

If you do not know where you want the students to be by the end of the semester, there is little chance that they will get there. As you set your goals, you need to be optimistic. I have found that students respond well to the challenge to work hard provided that they believe that your goals are attainable and that the necessary support mechanisms are in place. Unrealistic goals can be dangerous. Once students begin to feel overwhelmed, they adopt a strategy that David Tall [5] has called *disjunctive generalization*. They jettison the search for understanding and switch to memorization, regardless of contradictions or inconsistencies, as the safest route to a passing grade.

Writing down your goals for the course is a good exercise for you. Think about the kinds of problems you want students to be able to solve by the midpoint, by the end of the course, a year from now. How important are the ability to apply this mathematics to novel situations? to analyze the components of the theory they have learned? to creatively synthesize these components into creative problem solving? What are the big ideas that they need to be conversant with? What are the connections that they have to be able to make?

If you want to share this list of goals with your students, that is fine. It will help relieve some of their start-of-semester anxiety. But you communicate these goals in how you choose to assess their performance. If you do not measure and reward their progress, then few students will make those goals their own.

3 Use assessment effectively

It is common to base most of the course grade on student performance in two mid-term exams and a final. Students are accustomed to exam questions that are a mixture of routine straight-out-of-the-book exercises and a few more challenging problems. They know that they can get a good grade by practicing homework problems until they have the basic patterns memorized. They do not have to be able to answer the most difficult problems. We may say that our goal is for our students to understand the mathematics, but if we use this standard format to assess their knowledge, then we are telling them that what we really care about is whether they can mimic the template solutions quickly and accurately. We are allowing them to bypass understanding.

We want our students to be able to do basic calculations, but we also want them to learn more than this. We would like to be able to replace the standard exam with one that consists of probing, challenging questions that give students a chance to show off what they really know. I know from my own experience that unless I have specifically prepared my students for such a test, the result is abysmal scores and vociferous complaints that the test was unfair; it was not the kind of test my students were expecting. If we have given a challenging test on which students have scored poorly, we are tempted to use a curve to determine

grades. This undermines what we were trying to accomplish because it confirms the message that partial credit is good enough and real understanding is not important.

The standard assessment based on two mid-terms and a final is a trap that leads to one of three unsatisfactory outcomes: either students see facility with certain well-defined procedures as the goal of the course, or they learn to rely on a curve and so know that they will not be held accountable for any real understanding, or most of the students fail, which not only makes life difficult for the math department but shows that you are not teaching *your* students.

There are other ways of approaching assessment.

Assessment is the carrot and stick that you can use to shape student attitudes and study habits and to communicate what you want students to learn from your course. If you want students to read the text, then give unannounced quizzes on the readings that they were to have done. If you want students to reflect on what they have learned and think about what is happening in the processes that they are mastering, then have them keep a journal that is periodically graded or have them write reports that require this reflective approach. If you want your students to be able to apply their knowledge to unfamiliar situations, then give them problems and projects that require this level of understanding. If you want your students to listen critically to your lectures and be able to distill the main points, then ask them to write these down at the end of the class and hand them in. If you want students to be able to use definitions and theorems correctly and unambiguously, then have them write assignments where this is required. If there are basic skills that you want students to master, then test these skills and set the bar for a passing grade as high as you feel is needed, whether that be 80%, 90%, or 100%.

Assess early; assess often. In my experience, students react positively to this. They appreciate the feedback and direction that it gives them. It reduces the stress of a major examination that counts for a third or more of their grade and for which they are not quite sure what will be expected. Students should never be surprised by what you expect of them. You should be shaping their approach to the class from day one. There is nothing wrong with putting probing, challenging questions on your examinations provided this is nothing new and that students have been given the means to tackle such questions. There lies the nub of the difficulty. Once you know where your students are as they begin the class and you have made clear to them what will be expected, how do you make it possible for them to achieve those goals?

4 Put supports in place

Take your big goals and look at the pieces that have to be in place in order to achieve them. You want your students to be able to tackle and solve an unfamiliar problem. It requires going back through all that they have seen this

semester, picking out appropriate ideas, and putting them together in what is for them an original configuration. Do they need to be conversant with the big ideas of the class? Do they need to be able to go back and reread sections of the text? Do they need to be certain about the meaning and significance of certain theorems? Do they need to be able to express their own understandings clearly? If so, then these are things that you should have been emphasizing and assessing all along.

You may need to devote time to how to read a math book. Most students believe that this is not possible, so just testing them on how well they read will not be enough. You may need to spend time talking about what you expect in technical reports. You may need to talk about techniques of problem solving. If students know that these are skills for which they will be evaluated, they will pay attention. If your syllabus is so crammed that there is no space to work on the skills you consider to be important, then there is something wrong with your syllabus.

The most important support that can be put in place is the opportunity to practice the high level skills that you will demand. This should include critical feedback and provision for students to redo the assignment. One of the great drawbacks of the traditional "two mid-terms and a final" model of assessment is that it re-enforces the perception that the way to get through classes is to take the week before an exam, focus all energy on that course, clear that hurdle, and then forget all that has been learned to clear memory space for the next hurdle. Traditional assessment re-enforces the attitude that success in mathematics is the result of natural aptitude and not something that can be cultivated and developed. We know that mastery comes through the process of attempting and failing and then going back to find the roots of that failure and correcting them. This is an attitude toward learning that we would hope that our students know before they arrive in our classes, but they usually do not. It is our responsibility to lead them through this process until it becomes their own.

I do this through projects and the reports that students write. I have a clear set of criteria about the level of precision and clarity that I expect. Incorrect mathematics, inadequate explanations, and poor writing are all critiqued in the first submitted version which is returned to the students for reworking. On mid-term examinations, after I grade and return the test, my students have several days in which to correct the answers that they missed. They can regain some of the points that they lost if they can show that they can now solve that problem. The only complaint I ever received about this policy was from a student who did not want to be forced to think about the test questions he had missed after the exam was over. That complaint has become one of my primary justifications for this policy.

5 Make students active participants

At one of my first national meetings, I picked up a button that proclaimed, "Math is not a spectator sport." We all learn by doing. I do not know any responsible teachers who do not want students to wrestle with the mathematics that we have explained to them and to practice the higher level thinking skills in which we want them to excel. The problem is that most of our students have no idea how to begin to interact with mathematics in this way. For them, practicing mathematics outside of class is doing the even exercises numbered 2 through 20 at the end of the section, and nothing more. Most of our students come into our classes with no conception of how to begin tackling an unfamiliar problem, or what to do if the first line of attack fails. The answer is not to despair of who they are, but to help them become the students that we want them to be. Our opportunity for shaping the behaviors that we want them to adopt is in the classroom.

Some of the most effective learning I have witnessed has been in group situations in the classroom where a small group of two or three or four students is tackling a challenging and unfamiliar problem, a problem that they will then carry out of the classroom to continue working on. The group dynamics are important. Students working on their own are more likely to freeze and try nothing if the correct approach is not immediately apparent. They are more likely to stick to an unproductive strategy despite its futility. They are less likely to see alternate procedures that might simplify or simply clarify what has worked. As I have seen repeatedly, a small group of students collectively can solve a problem that none of them individually could have worked out. The result is increased confidence and experience in lateral thinking.

Lecture is still one of my tools for teaching, but I have learned that it is most effective when broken up by opportunities for students to actively engage the topic that I am explaining. This includes asking probing questions and giving students time to think about or work on the answers. When there is doubt or hesitancy about the answer, I ask several students to put the answer into their own words. Where there is divided opinion about the correct answer, it is helpful to stop the class and have students discuss it with those around them. A tremendous amount of learning transpires when a student has to explain his or her own understanding to someone else.

6 Encourage group work

Few of the catchwords associated with the reform movement in mathematics instruction have been as controversial as collaborative or cooperative learning. I am a believer for the reasons given above, because I have had students tell me that this is where they learned how to analyze unfamiliar problems, and because of the role that it plays in developing study groups and support networks.

I have held exit interviews with graduating seniors. One of the consistent factors cited as important for their success in college was learning to form and make use of study groups. This was not something that they knew would be helpful when they came to college. Their first study groups grew out of the interactions with classmates that were created by group projects.

There are many approaches to collaborative or cooperative learning. My advice is to talk with others who have experience and then experiment with what feels comfortable to you. The fact that students will brainstorm a problem collectively does not mean that they have to receive a common grade. I have often had each member of the group write up his or her own report. But group grades are useful early in the semester to force students to learn to work together.

7 Use technology as appropriate

Technology is the other controversial catchword of the reform movement. This is the one place where there really is something new under the sun, but reformers are far from unanimous in their understanding of what it means for teaching or how it should be used. My own advice is to stay informed and be willing to experiment with the ideas and approaches that make sense to you. There have been many failures. There also have been many successes. We are still in the early stages of learning how to use this tool.

Advocates of technology say that it enables students to focus on the ideas rather than rote manipulations. Critics assert that it becomes a crutch and that students who are not fluent with fundamental processes are handicapped when approaching higher level problems. My own experience is that both sides are correct, and the important and difficult question is where to draw the line. At what point do we introduce, permit, or encourage the use of technological tools? The answer is highly dependent on our immediate objective. When I want my calculus students to discover the orthogonality of the sine and cosine functions of various frequencies, it is essential to use a computer algebra system to ensure speed and accuracy in performing integration by parts. But before we begin such an exercise, my students will have done a lot of integration by parts problems by hand because I believe that it is a fundamental skill. Its mastery is essential to the appreciation of its consequences.

There are many different ways in which computers or graphing calculators can be incorporated into classes. They can be used to prepare demonstrations, but be wary of presentations that are too slick. Remember that your job is not to entertain but to get students to think. Many math classes now incorporate labs where computers are used for numerical calculations, or to aid in visualization, or for work at the symbolic level. What I have found through painful personal experience is that any laboratory must be tightly integrated into what is happening in the classroom, reinforcing what happened in the previous class and preparing for the next. When the laboratory experience is well thought

through, it can provide powerful reinforcement of the lessons that you want to communicate. When it is poorly conceived, it is nothing more than a frustrating waste of time for you and your students.

8 Be open to curricular reform

When I have thought long and hard about a course—what works and what does not work for my students—I find myself dissatisfied with the traditional syllabus and the available textbooks. The fact that I am not alone is reflected in the myriad reform curricula and textbooks. This is not a monolithic movement. It is characterized by wide diversity. In sorting through what is available, you need to be aware of the goals and priorities of those who developed the materials.

My own primary criterion in choosing or developing curricular materials is to have a driving theme that generates questions what will puzzle, discomfort, and challenge my students. I have come to appreciate that the pure Euclidean ideal of finished mathematics is not appropriate for most teaching. It is a polished surface too slippery for most students to grasp. Imre Lakatos [3, page 140] has gone so far as to accuse Euclid of being "the evil genius particularly for the history of mathematics and for the teaching of mathematics." A kinder assessment is given by David Tall in his "Reflections" on *Advanced Mathematical Thinking* [6],

> This does not remove the need to pass on information in the theorem-proof-application mode, for this is the crowning glory of advanced mathematics. But students need to be assisted through a transition to a stage where they see the necessity and economy of such an approach.

Some of my own thoughts on how to assist students through this transition are expressed in my review of Serge Lang's *Undergraduate Analysis* [1] and in "True Grit in Real Analysis" [2].

If we want to change what students take from our courses, then we must change what we do. If nothing changes, then nothing changes. We must be realistic about where we and our students start, clear about what we want to accomplish, knowledgeable about how our students learn, and willing to experiment with our teaching to make it as effective as possible.

References

[1] David M. Bressoud, Review of *Undergraduate Analysis: Second Edition*, *The Mathematical Intelligencer*, **20** (1998), 76–77.

[2] ———, True Grit in Real Analysis, submitted to *PRIMUS*.

[3] Imre Lakatos, *Proofs and Refutations: The Logic of Mathematical Discovery*, Cambridge University Press, 1976.

[4] Thomas M. Sherman et al, The Quest for Excellence in University Teaching, *J of Higher Education*, **48** (1987), 66–84.

[5] David Tall, The Psychology of Advanced Mathematical Thinking, pages 3–21 in *Advanced Mathematical Thinking*, David Tall ed., Mathematics Education Library, vol. 11, Kluwer Academic Publishers, Dordrecht, 1991.

[6] ———, Reflections, pages 251–259 in *Advanced Mathematical Thinking*, David Tall ed., Mathematics Education Library, vol. 11, Kluwer Academic Publishers, Dordrecht, 1991.

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